Event-triggered Model Predictive Control for Autonomous Vehicle with Rear Steering

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Presentation Outline

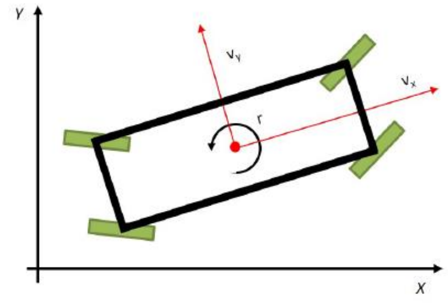
- Background and Motivation
- Problem Formulation
- Event-triggered Model Predictive Control
- Numerical Simulation Results
- Conclusion

Background

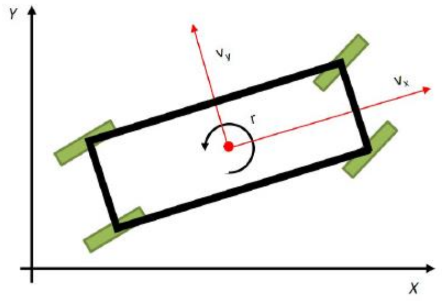
- In steer-by-wire system, the mechanical connection between the steering wheel and road wheels is replaced by electronics, algorithms, and actuators.
- The use of electronic control system allows much more precise control.
- It also allows active steering control where the driver’s command may be intelligently altered.
- The disadvantage includes potential delay in control systems and the lack of “road feel”.

Background

- Rear steering capability has been recently introduced by OEM to increase vehicle agility and stability.
- For passive rear steering, the rear wheel is programmed to be
  - Out of phase with the front wheel in low speed to increase agility
  - In phase with the front wheel in high speed to increase stability.
  - The ratio between rear wheel angles and front wheel angles is fixed.



Out of phase ARS



In phase ARS

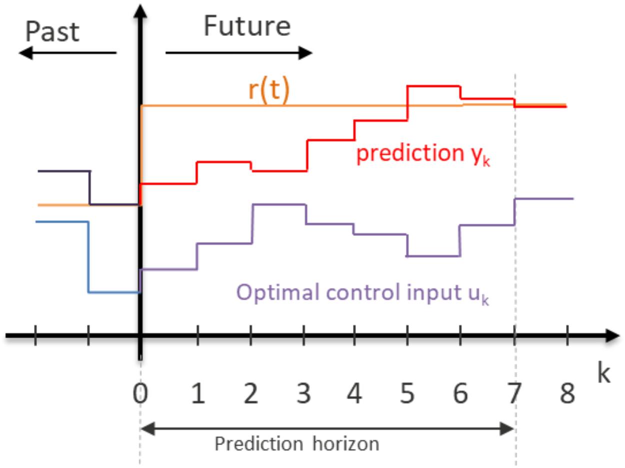
Background

- For active rear steering, the rear wheel angles are computed in real-time.
- Active rear steering can be used for human driver or autonomous vehicles (AV).
- For its real-time optimal control, model predictive control (MPC) has been investigated.
- However, as active rear steering increase the vehicle flexibility, the number of optimization variables is also increased.
- And the high computational requirement of MPC prevents its usage for massive production.

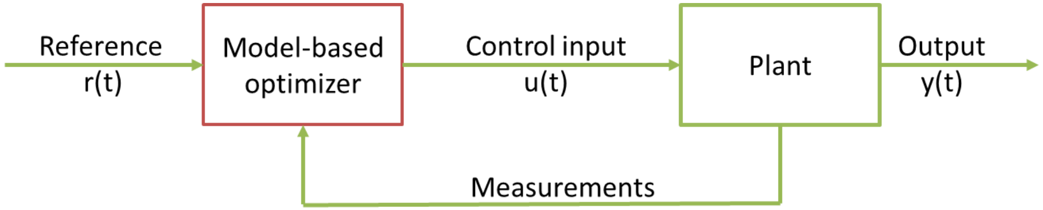
MPC Formulation

Model Predictive Control (MPC)

- solves a model-based constrained optimization in real-time,
- finds the optimal control sequence over a finite horizon,
- applies only the first optimal control action to actuators,
- repeats above optimization process is at new time step with new measurement.



$$
\begin{aligned}
\min_{Z_t, U_t} \quad & J(Z_t, U_t) \\
\text{s.t.} \quad & \zeta_t = \hat{\zeta}_t \\
& \zeta_{t+k} = f(\zeta_{t+k-1}, u_{t+k-1}), \quad 1 \le k \le p \\
& \zeta_{min} \le \zeta_{t+k} \le \zeta_{max}, \quad 1 \le k \le p \\
& u_{min} \le u_{t+k} \le u_{max}, \quad 0 \le k \le p-1 \\
& \Delta_{min} \le u_{t+k} - u_{t+k-1} \le \Delta_{max}, \\
& \qquad\qquad 0 \le k \le p-1,
\end{aligned}
$$



MPC Formulation

- We consider the autonomous vehicle (AV) path following problem.
- The vehicle is assumed to have four-wheel-steering capability.
- MPC is to optimize both front and rear steering angles.

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\begin{aligned}
\min_{Z_t, U_t} \quad & J(Z_t, U_t) \\
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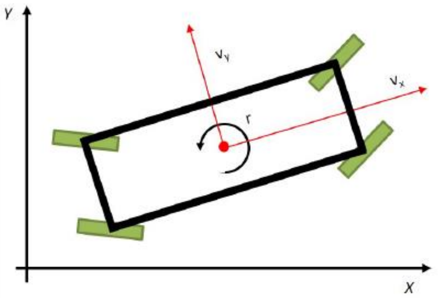
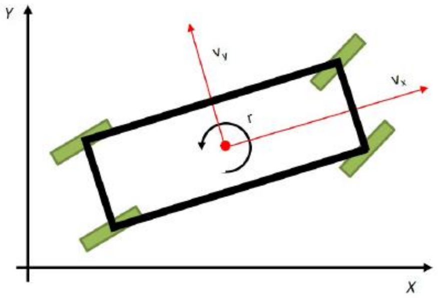
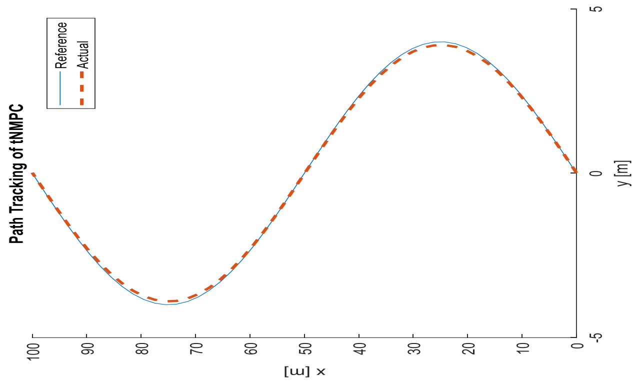


Out of phase ARS



In phase ARS



MPC Formulation: Bicycle Model

- To reduce computation, we use bicycle
- 6 degree-of-freedom planar model with longitudinal, lateral and yaw dynamics,
- In addition, linear tire force model, aero dynamics and wheel dynamics are also included.
  - Load transfer is ignored as we only considered x-y planar model.

$$
\begin{aligned}
\min_{Z_t, U_t} \quad & J(Z_t, U_t) \\
\text{s.t.} \quad & \zeta_t = \hat{\zeta}_t \\
& \boxed{\zeta_{t+k} = f(\zeta_{t+k-1}, u_{t+k-1}), \quad 1 \le k \le p} \\
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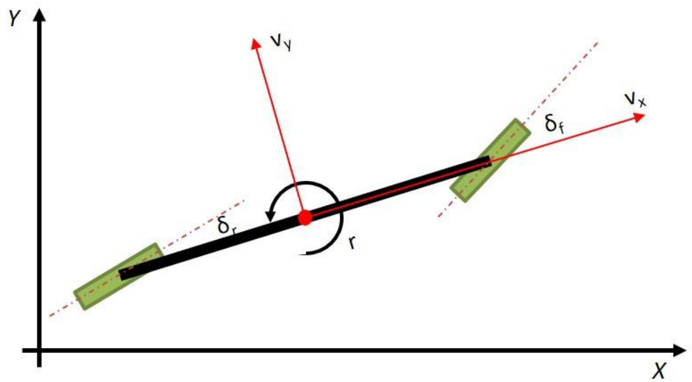
$$



MPC Formulation: Bicycle Model

- x, y, and ψ are in global coordinate.
- v_x, v_y, and r are in vehicle coordinate.
- δ_f and δ_r are front and rear steering angles.

$$\begin{aligned} \min_{Z_t, U_t} \quad & J(Z_t, U_t) \\ \text{s.t.} \quad & \zeta_t = \hat{\zeta}_t \\ & \zeta_{t+k} = f(\zeta_{t+k-1}, u_{t+k-1}), \quad 1 \le k \le p \\ & \zeta_{min} \le \zeta_{t+k} \le \zeta_{max}, \quad 1 \le k \le p \\ & u_{min} \le u_{t+k} \le u_{max}, \quad 0 \le k \le p-1 \\ & \Delta_{min} \le u_{t+k} - u_{t+k-1} \le \Delta_{max}, \\ & \qquad 0 \le k \le p-1, \end{aligned}$$

Longitudinal position $$\dot{x} = v_x \cos\psi - v_y \sin\psi$$

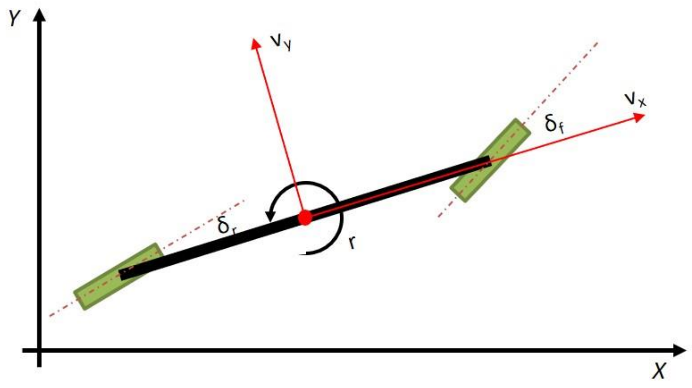
Longitudinal velocity $$\dot{v}_x = v_y r + \frac{2}{m} \sum_{i=f,r} F_{x,i} - g \sin\sigma_g - \frac{1}{m} F_a$$

Lateral position $$\dot{y} = v_x \sin\psi + v_y \cos\psi$$

Lateral velocity $$\dot{v}_y = -v_x r + \frac{2}{m} \sum_{i=f,r} F_{y,i}$$

Heading angle $$\dot{\psi} = r$$

Yaw rate $$\dot{r} = \frac{1}{I} \left(2L_{xf} F_{y,f} - 2L_{xr} F_{y,r} \right),$$



MPC Formulation: Cost Function

- As demonstrate, the vehicle is to follow a sinusoidal path.

$$\begin{aligned} \min_{Z_t, U_t} \quad & J(Z_t, U_t) \\ \text{s.t.} \quad & \zeta_t = \hat{\zeta}_t \\ & \zeta_{t+k} = f(\zeta_{t+k-1}, u_{t+k-1}), \quad 1 \le k \le p \\ & \zeta_{min} \le \zeta_{t+k} \le \zeta_{max}, \quad 1 \le k \le p \\ & u_{min} \le u_{t+k} \le u_{max}, \quad 0 \le k \le p-1 \\ & \Delta_{min} \le u_{t+k} - u_{t+k-1} \le \Delta_{max}, \\ & \qquad 0 \le k \le p-1, \end{aligned}$$

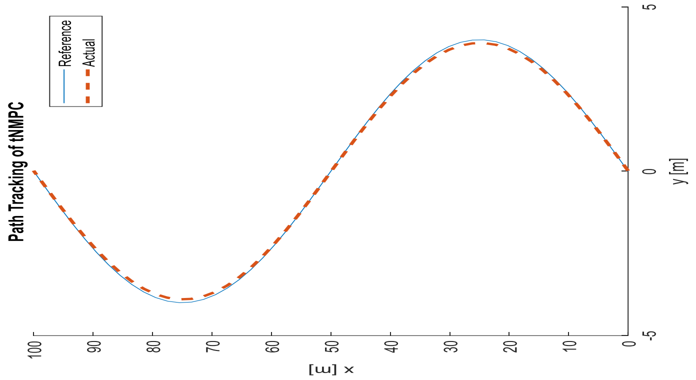
Excessive use of rear steering

Steering busy-ness

$$J(Z_t, U_t) = \sum_{k=0}^{p-1} \left(\| u_{t+k} - u^r_{t+k} \|^2_{Q_u} + \| u_{t+k} - u_{t+k-1} \|^2_{Q_d} \right)$$

$$+ \sum_{k=1}^{p} \left\| \zeta_{t+k}(3) - 4\sin\left(\frac{2\pi}{100} \zeta_{t+k}(1)\right) \right\|^2_{Q_t}$$

Path tracking error



MPC Challenge

- For conventional MPC, the optimization is repeated at every sampling time step.
- For steering application, a sampling time of 1 second is often adopted.
- The nonlinear MPC formulated above requires high computing power that may not be available in production grade ECU.

$$
\begin{aligned}
\min_{Z_t, U_t} \quad & J(Z_t, U_t) \\
\text{s.t.} \quad & \zeta_t = \hat{\zeta}_t \\
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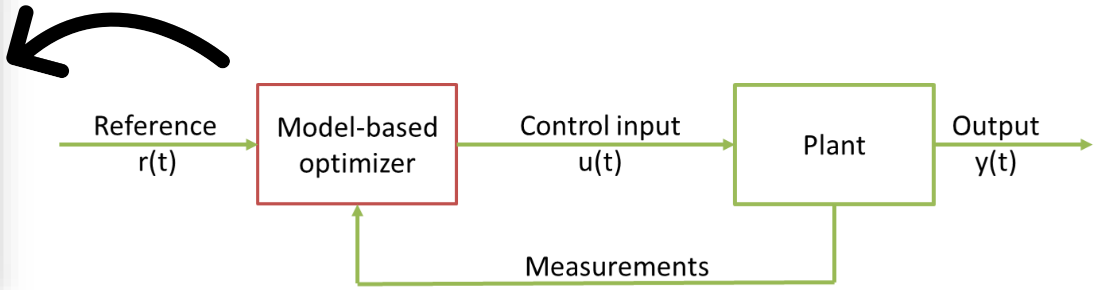
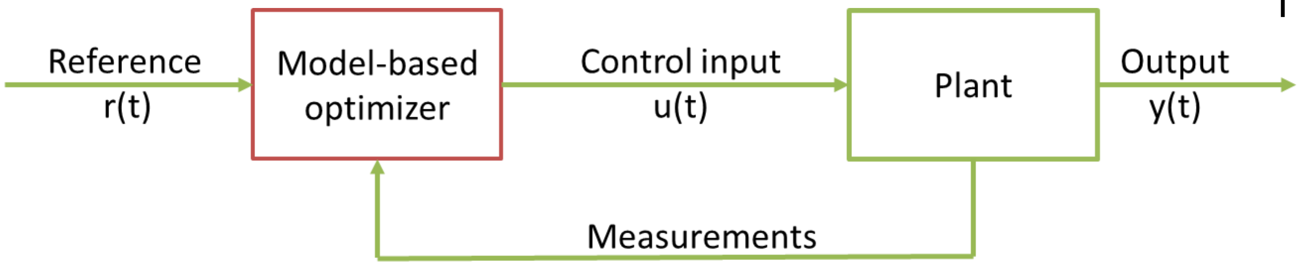
$$



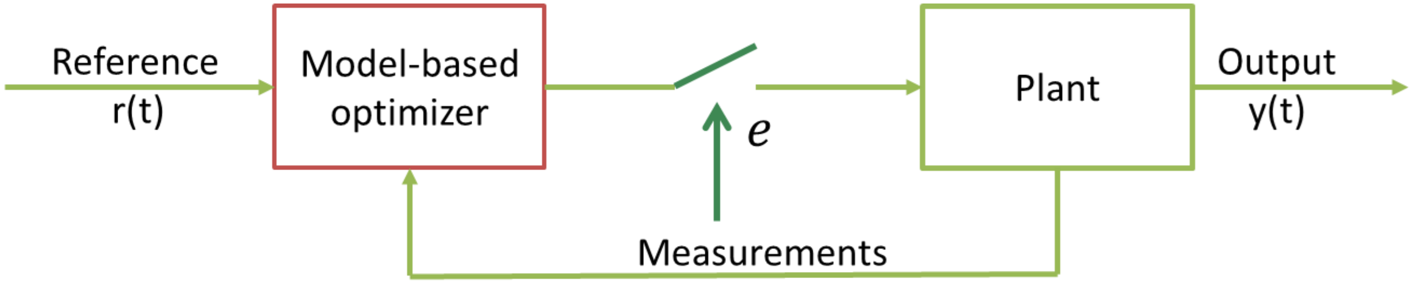
Event-Triggered MPC

- Event-triggered MPC reduces computational requirement by solving optimization problem on demand.

Time-triggered MPC

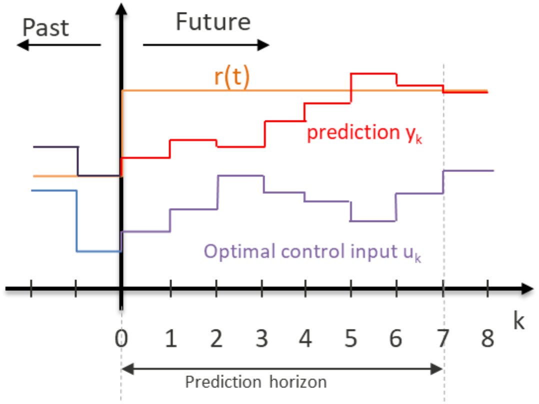


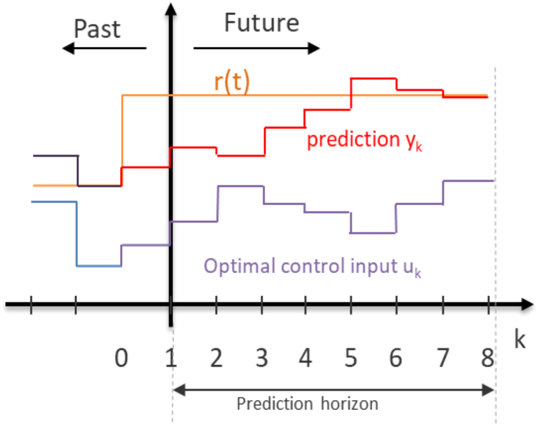
Event-triggered MPC



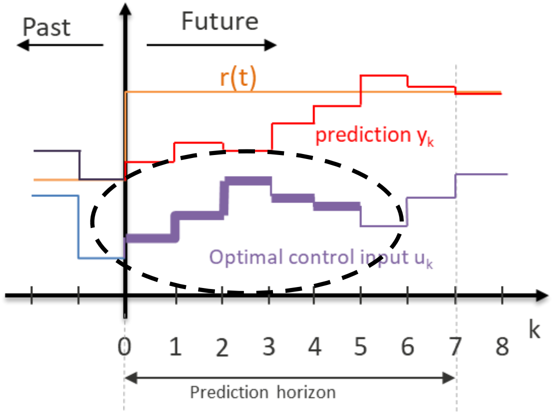
Event-Triggered MPC

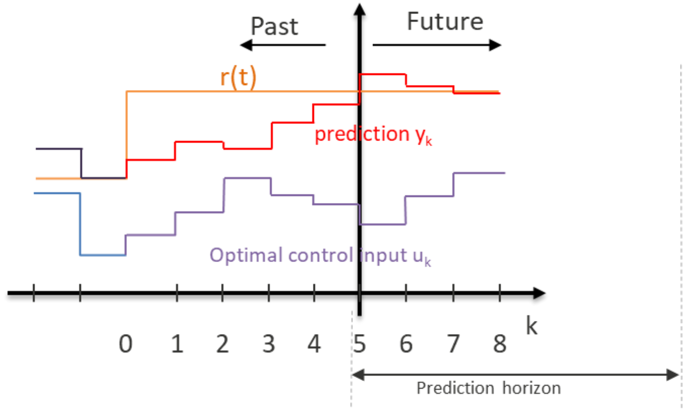
Time-triggered MPC





Event-triggered MPC





Event-Triggered MPC

- Time-triggered MPC solves the optimization at fixed sampling time, implements the first elements of optimal control sequence, and abandons the rest.
- Event-triggered MPC solves the optimization only if a triggering event is on, defined as:

$$e = \begin{cases} 1 & \text{if } ||Z_{t_1}(k) - \hat{\zeta}_t||_Q^\infty > \sigma \text{ or } k > k_{max} \\ 0 & \text{Otherwise} \end{cases}$$

- In this case, the first elements of optimal control sequence is implemented, and the rest is passed to the next control loop.
- When no triggering event, the previous optimal sequence is shifted to obtain control action

$$u = \begin{cases} \text{Solution of } (2) & \text{if} \quad e = 1 \\ U_{t_1}(k+1) & \text{Otherwise} \end{cases}$$

Algorithm 1 Event-Triggered NMPC

procedure ENMPC($\hat{\zeta}_t$, k, U_{t_1}, Z_{t_1})
  $k \leftarrow k + 1$;
  $e \leftarrow$ computing (5);
  if $e = 1$ **then**
    $k \leftarrow 0$;
    $(Z_t, U_t) \leftarrow$ Solving OCP
    $u \leftarrow U_t(1)$;
    $U_{t_1} \leftarrow U_t$;
    $Z_{t_1} \leftarrow Z_t$;
  else
    $u \leftarrow U_{t_1}(k+1)$;
  end if
  return u, k, U_{t_1}, Z_{t_1}
end procedure

Numerical Simulation Results

- Model mismatch is introduced in the simulation environment to test control robustness.
- Time-triggered MPC and event-triggered MPC use different calibration for the cost function.
- Input constraints are used to further reduced the abrupt change of steering angle.

Table 2. Parameters for The Bicycle Model

Parameter [Unit]	NMPC	Virtual Vehicle
M [kg]	1500	1425
L_{xf} [m]	1.2	1.3
L_{xr} [m]	1.4	1.3
I [kgm^2]	4192	4402
R [m]	0.2159	0.2159
C_i [-]	-4.5837	-4.5837
μ_i [-]	1	0.95

Table 3. MPC Calibrations

Calibration	tNMPC	eNMPC
Q_t	20	20
Q_u	[30,0;0,60]	[10,0;0,45]
Q_d	[50,0;0,6.8]	[100,0;0,20]
Q	-	[25 0;0 20]
σ	-	1

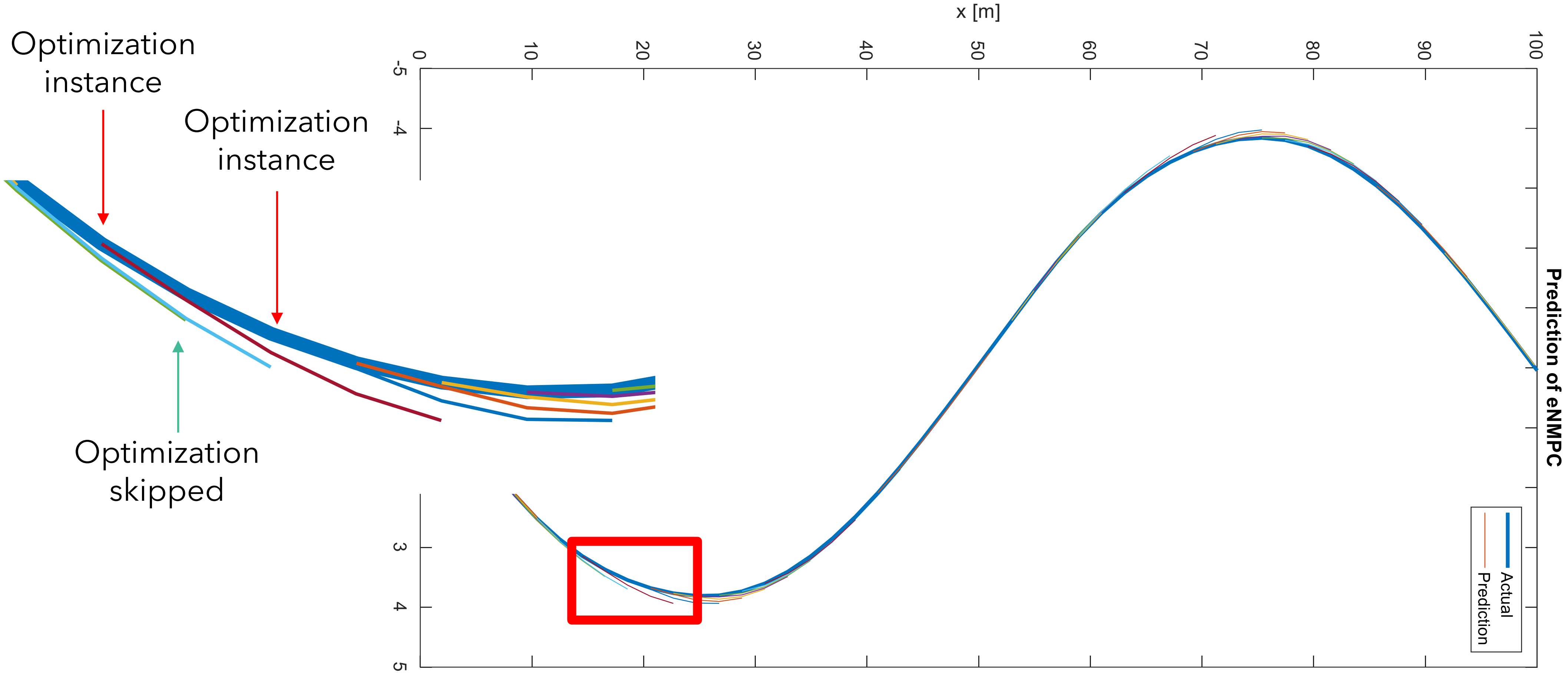
$$u_{max} = \begin{bmatrix} 0.54105 \\ 0.17453 \end{bmatrix}$$

$$u_{min} = \begin{bmatrix} -0.54105 \\ -0.17453 \end{bmatrix}$$

$$\Delta_{max} = \begin{bmatrix} 0.034907 \\ 0.034907 \end{bmatrix}$$

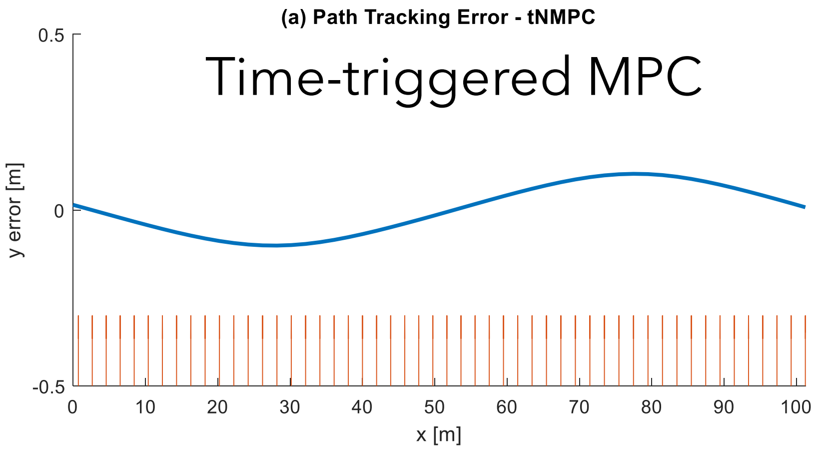
$$\Delta_{min} = \begin{bmatrix} -0.034907 \\ -0.034907 \end{bmatrix}$$

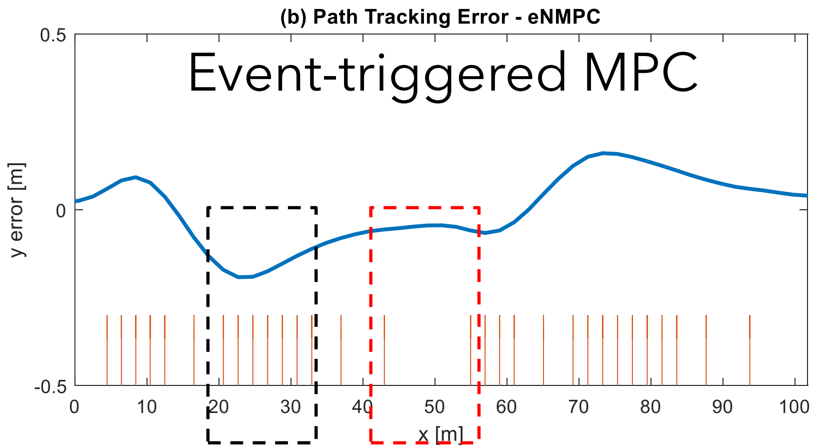
Numerical Simulation Results



Numerical Simulation Results

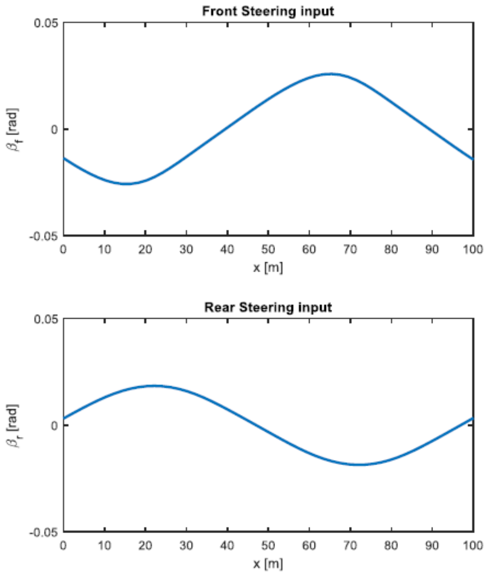
- Event-triggered MPC achieves similar path tracking error compared to conventional time-triggered MPC.
- Event-triggered MPC saves up to 50% computations, significantly relaxing the requirement on ECU computing power.



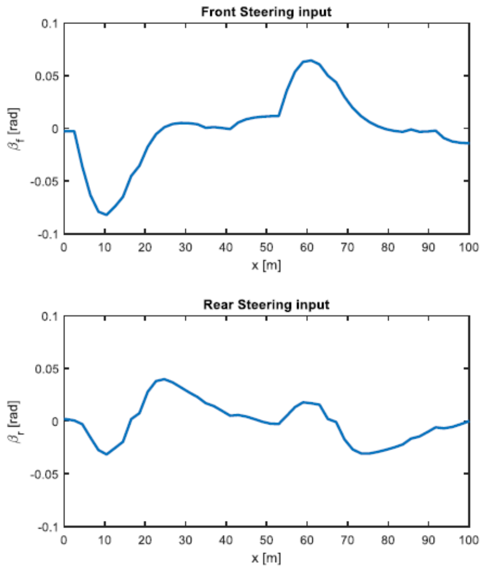


Numerical Simulation Results

- Time-triggered MPC always use out-of-phase steering, while event-triggered MPC uses both out-of-phase and in-phase.
- Event-triggered MPC results less smooth control commands.
- Event-triggered MPC relies more on rear steering.
- The impacts on ride comfort deserves future investigation!



Time-triggered MPC



Event-triggered MPC

Conclusion

- Active rear steering increases the control flexibility, while at the same time requires higher computing power for its real-time optimal control.
- The proposed event-triggered MPC formulation can significantly lower the computing requirement, and maintains comparable control performance.
- As future work, the impact on ride comfort will be investigated, by penalizing large lateral acceleration in the cost function.

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