Extended Kalman filter steady gain scheduling using k-means clustering

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Abstract: This paper studies the gain scheduling problem for extended Kalman filter (EKF). To save throughput, the steady gain is usually used for Kalman Filter. In the context of EKF, there is no universal steady gain. In this paper, we propose a methodology to schedule the steady gain for EKF. The idea is to offline linearise the nonlinear model at various operating points, and for each of the linearised systems, to compute the steady gain corresponding to conventional Kalman filter by solving the corresponding algebraic Riccati equation. The operating space is then divided into multiple zones, through k-means clustering algorithm, so that within zone, the steady state gains are close to each other. For real time filtering, the centroid of each zone is used as gain for correction step, instead of computing the time-varying gain online, hence saving throughput. We demonstrate the proposed methodology in a two-state nonlinear system.

Keywords: state estimation; EKF; extended Kalman filter; steady gain; k-means; gain scheduling, throughput, embedded computation.

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1 Introduction

To improve the applicability of Kalman filter, one practice that is often used to save throughput is to schedule the gains, which would normally be calculated online in the conventional algorithm. It is then well known that the finer the gain scheduling is, the better results the filter can provide. For example, ref (Hollander et al., 1968) studies the gain schedule of a 15-state linear Kalman filter for inertial navigation system. Closed form expression for Kalman gain is proposed in Crotteau (1969), by fitting the Kalman gain trajectories (if computed online) to a curve. The mathematical concept on how to schedule gains for linear regulator problem is studied in Kleinman and Athans (1968) and Kleinman et al. (1968). Though different from state estimation problem, these studies provide a theoretical framework that can also be adopted for Kalman filter gain scheduling.

More recently, extended Kalman filter (EKF) has attracted a lot of attention in both academia and industries, due to its capability to handle nonlinear system dynamics (Julier and Uhlmann, 2004; Wang and Papageorgiou, 2005; Reif et al., 1999; Kim et al., 1994; Hoshiya and Saito, 1984; Sabatini, 2006; Pham et al., 1998; Wan and Van Der Merwe, 2000; Song and Speyer, 1985; Lee and Ricker, 1994). And its gain scheduling has been studied in Kobayashi et al. (2005), Andersson (2005) and Yoo et al. (2011), for various industrial applications, such as aerospace, vehicle, engine control, etc. Note that different from steady EKF, or constant gain EKF, the gain scheduling EKF has the capability to use different gain according to the operating points, though within small range the gain remains constant. One commonality of these studies is to fit the gain to a smooth function, which can then be scheduled online without additional complexity. However, such approach requires the practitioner to have very thorough domain knowledge in order to choose the best function/space to fit the gain. Another drawback of these approaches is that the gain for a specific operating point is obtained by fine tuning, hence lack of theoretic foundation.

In this paper, we propose a methodology to schedule gain for EKF. The idea is to offline linearise the nonlinear model at various operating points, and for each of the linearised systems, to compute the steady state gain corresponding to conventional Kalman filter by solving a algebraic Riccati equation (hence different from the aforementioned work). The operating space of the nonlinear model is then divided into multiple zones, through k-means clustering algorithm, so that within zone, the steady state gains are close to each other. Note that these steps are performed offline, hence without adding computing requirement for online filtering. For real time filtering, the centroid of each zone is used as gain for correction step, instead of computing the time-varying gain online, hence saving throughput. The results obtained through a two-state nonlinear system show the throughput saving without impacting filtering performance.

Comparing to literature, the methodology proposed in this paper allows EKF gain scheduling based on the characteristics of the linearised model, as opposed to manual scheduling that are widely used in literature. Furthermore, the proposed method only incurs minimal throughput increase, allowing real-time implementation in embedded systems.

The remainder of this paper is organised as follows. Section 2 presents background information on EKF and k-means clustering algorithm. Our proposed algorithm is presented in Section 3, with Section 4 shows numerical results on a two-state nonlinear systems. The paper is concluded with Section 5.

2 Background

2.1 Extended Kalman filter

Consider the following nonlinear systems

$$x_{k+1} = f_k(x_k, u_k) + w_k \tag{1}$$

$$y_k = h_k(x_k) + v_k, \tag{2}$$

where u, x, and y denote the input, state, and output of the system. w is the process noise and v the measurement noise, with covariance matrices Q and R respectively. Without loss of generosity, Q and R are diagonal matrices. As an operating point (u, x), denote the corresponding linearised time invariant systems as

$$x_{k+1} = Ax_k + Bu_k + w_k \tag{3}$$

$$y_k = Cx_k + v_k. \tag{4}$$

For simplicity of notation, the time index k for A, B and C are omitted. Furthermore, we define the following operators:

$$\cdot := \cdot(k|k)$$
$$\cdot = \cdot(k|k-1)$$

In other words, $\cdot$ is the *a posteriori* estimation and $\cdot$ is the *a priori* estimation, of the corresponding variable.

Given state estimation $x := x(k|k-1)$, covariance matrix $P := P(k|k-1)$, and measurement y, the measurement update of EKF is as follows

$$x = x + K(y - h_k(x)) \tag{5}$$

$$K = PC^T(CPC^T + R)^{-1} \tag{6}$$

$$P = P - K(PC^T)^T \tag{7}$$

Given state estimation $x := x(k|k)$ and covariance matrix $P := P(k|k)$, the time update of EKF is as follows

$$x = f_k(x, u_k) \tag{8}$$

$$P = APA^T + Q \tag{9}$$

Note that with abuse of notation, x and P for time update correspond to one step prediction at time k, i.e., $x = x(k+1|k)$ and $P = P(k+1|k)$.

Note that to implement the conventional EKF, the following online computations are necessary for each time step

- online linearisation of equations (1) and (2) to obtain matrices A, B, ad C for equations (3) and (4)
- prediction using equation (8) and correction using equation (5)
- computing Kalman gain for correction step according to equation (6)
- updating covariance matrix in both time and measurement updates, according to equations (7) and (9).

Note that the gain scheduling for EKF usually aims to eliminate the online computation for calculating K and updating P, i.e., equations (6), (7), and (9).

2.2 k-means clustering

Given a set of observations $\mu_1, \ldots, \mu_n$, k-means (Lloyd, 1957) aims to find k clusters (or groups) such that a point is considered to be in a particular cluster if it is closer to that cluster's center (called centroid) than any other centroid. The classical k-means algorithm finds the best clustering by iteration, and at each iteration, the algorithm

- assigns data points to clusters based on the centroids of current iteration
- update centroids according to the points of the newly formed cluster.

The algorithm terminates when the cluster membership does not change, or the preset maximum allowable iteration is hit. Note that for k-means, the number of clusters k, is an input parameter that must be specified.

3 EKF gain scheduling

Recall that the EKF gain scheduling problem considered in this paper aims to eliminate the online computation for calculating K and updating P, i.e., equations (6), (7), and (9). In order to do so, given a nonlinear system (1) and (2), the following steps are performed offline to computer the steady Kalman gain and cluster the input-state space (u, x) according to the steady gain:

- Generating N random operating points (u_k, x_k), and for each of the operating point, obtain the LTI system (3) and (4).
- For each of the LTI system obtained the corresponding steady state covariance matrix P by solving the Ricatti equation

  $$APA^T + Q = P - PC^T(CPC^T + R)^{-1}CP^T$$

  Once such P is found, the steady state Kalman gain K is computed according to

  $$K = PC^T(CPC^T + R)^{-1}$$

- Given the set of steady state Kalman gains $K_1, \ldots, K_N$, one for each LTI, perform k-means clustering to divide them into k clusters, and computed the centroid $K_{\mu,1}, \ldots, K_{\mu,k}$ for each of the cluster. The operating space (u, x) are also grouped according the clustering on K.

Upon the completion of the offline computation, the input-state space (u, x) is divided into k clusters, such that within each cluster, the steady Kalman gain is very similar, and can be represented by the centroid K_μ. Note that the number of random operating points N, and the number of clusters k, are both input parameters to the proposed algorithm, and are selected through calibration.

The following steps are then performed online for each time step,

- perform the time update according to equation (8)
- according to the pair of input u and current state estimate x, find the corresponding cluster and its centroid $K_{\mu,k}$
- upon arrival of new measurement y, perform the‘measurement update (5) with the scheduled $K_{\mu,k}$.

It can be easily noted that the new online computation, compared to the conventional one, save the throughput greatly, and hence suitable for embedded computing environment. However, since the steady gain is used, as can be seen in Section 4, there is some minor degradation in estimation performance, which is acceptable. Furthermore, since the covariance matrices P and P are no longer computed, the information concerning robustness of the estimated state x is no longer available.

4 Results

In this section, the proposed approach is applied to the following nonlinear systems

$$\begin{aligned} x_{1,k+1} =& x_{1,k} + T_s(k_1 x_{1,k}^3 + k_2 x_{1,k} - k_3 - x_{2,k} + k_4 u_k) \\ &+ w_{1,k} \end{aligned} \tag{10}$$

$$x_{2,k+1} = x_{2,k} + T_s a(b x_{1,k} - k_5 x_{2,k}^2) + w_{2,k} \tag{11}$$

$$y_k = x_{1,k} + v_k \tag{12}$$

with parameters in Table 1.

Table 1 List of model parameters for numerical analysis

Parameter	*Values*
k_1	–0.04
k_2	5
k_3	140
k_4	300
k_5	0.4
a	0.02
b	0.2
c	–50
T_s	0.01

For numerical analysis, the covariance matrices are selected as

$$Q = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}$$

$$R = 0.01$$

The number of gain clusters is selected to be three, and the number of random operating point is chosen as 10,000. Table 2 lists all the parameters that are used in algorithm simulation. Note that to assess the robustness of the proposed algorithm, the covariance matrices used for simulating w_k and v_k in the plant model are different from those used in EKF.

Table 2 List of algorithm parameters for numerical analysis

Parameter	*Values*
Q	[0.01, 0; 0, 0.01]
R	0.01
k	3
N	10,000

The clustering results are shown in Figures 1 and 2. In particular, this nonlinear system has 2 states and 1 measured output, and hence the gain matrix is a 2×1 matrix. Figure 1 plots the clustering results in the space of the elements of the gain matrix. Similarly, Figure 2 plots the corresponding clusters in the (u, x) space.

Figure 1 Clustering results of the steady state gain matrix (see online version for colours)

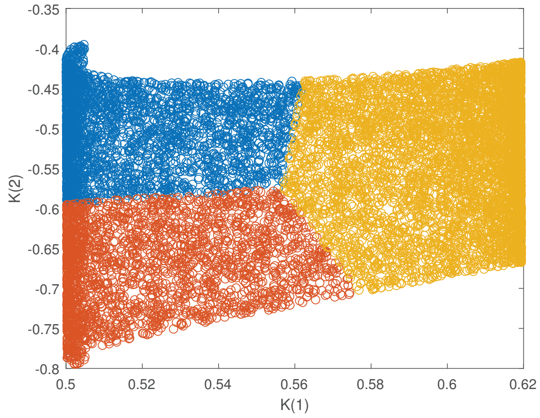


Figure 2 Clustering results of the operating space (u, x) (see online version for colours)

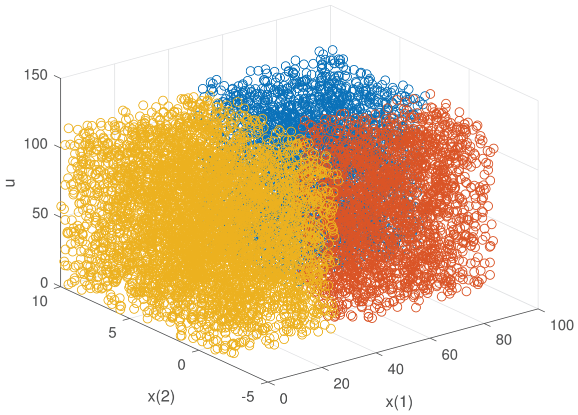
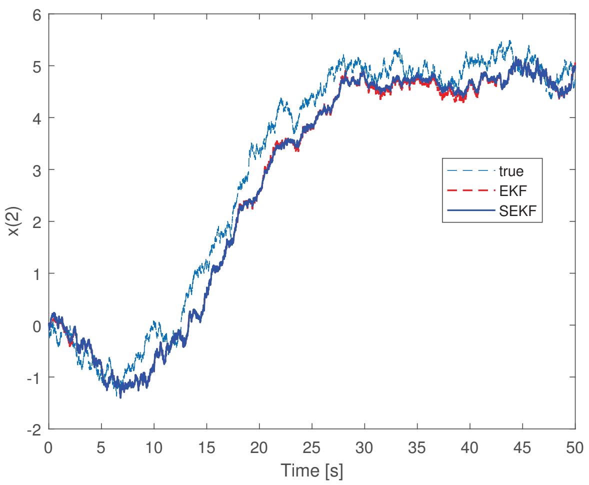


Figure 3 Comparison of estimated state $x(2)$: conventional EKF vs. proposed gain scheduled EKF (denoted as SEKF in figure) (see online version for colours)



The online state estimation performance of the proposed algorithm is presented in Figures 3–5. In particular, Figure 3 plots the true state x_2, the estimation obtained by conventional EKF using equations (5)-(9), and the estimation obtained by the proposed gain scheduling EKF. It is clear to see from Figure 3 that the propose approach gives very comparable results against the conventional EKF. Figure 4 shows a zoom-in version of Figure 3, further illustrating the performance of the proposed approach. Note that since in the proposed gain scheduling EKF, the steady state gain matrix is used, so there is some level of performance degradation as can be seen from Figure 4. However, the proposed approach avoids the various matrix multiplication that is normally required by conventional EKF in equations (6), (7) and (9), hence saving throughput for applications that have limitation in throughput.

Finally, Figure 5 plots the histogram of estimation error of x_2 for the proposed approach. It can be seen that, though using scheduled steady gain, the estimation error is well within reasonable range and does not have any bias.

Figure 4 A zoom-in version of Figure 3 (see online version for colours)

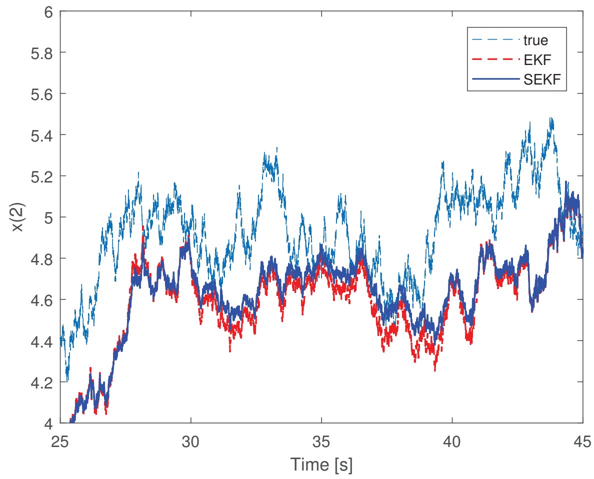
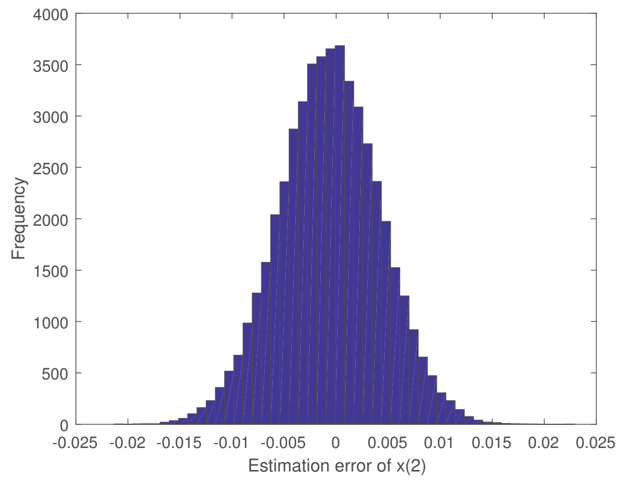


Figure 5 Histogram of estimated error of $x(2)$ by gain scheduled EKF (see online version for colours)



5 Conclusion

In this paper, a gain scheduling approach is proposed for EKF to avoid several matrix multiplications that are normally required by conventional EKF. The idea is to offline linearise the nonlinear model at various operating points, and for each of the linearised systems, to compute the steady state gain corresponding to conventional Kalman filter. The operating space of nonlinear model is then clustered into multiple zones through k-means clustering algorithm, so that within zone, the steady state gains are close to each other. For real time filtering, the centroid of each zone is used as gain for correction step, instead of computing the time-varying gain online, hence saving throughput. The numerical results by a two-state nonlinear system show great potential of the proposed approach, demonstrating by the comparable performance against the conventional EKF as well as reduced throughput. The future work will consider applying the proposed approach in a wider range of applications, such as parameter identification (Chen et al., 2017), energy systems (Chen and Rabiti, 2017), industrial reverse osmosis (Kim et al., 2016), failure diagnosis (Wang et al., 2019; Na et al., 2019), etc.

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